

Pure Azimuth Estimation Of Target Heading Using Single-Base Midpoint Direction Finding Formula

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Abstract:

In a two-dimensional plane, for a target in uniform linear motion, a stationary single station is assumed to be able to acquire the orientation information of the target in three consecutive equally spaced steps along the target's travelling path. Firstly, the trigonometric relationship is used to arrange the defining equation, from which the path difference between radial distances is obtained. Then the angle of arrival of the target relative to the detection station is determined with the help of the single-base midpoint direction finding formula, and then the heading angle of the target can be obtained from the relationship between the internal and external angles. The error analysis shows that the measurement accuracy obtained with the further help of single base mid-point direction finding method based on trigonometric fixed solution equations is better than that based on trigonometric method only.

Key words: Course angle; single fixed station; bearings-only location; DF; single base mid-point direction finding; passive location;

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I. Introduction

The problem of heading angle estimation has a very wide range of applications [1-5]. Existing theories of purely azimuthal target motion analysis prove that the heading of a uniformly moving target can be estimated by at least three or more purely azimuthal measurements from a stationary single station in a two-dimensional plane [6-9]. However, the fact that time covariates are included in all existing analyses, for this reason alone, the process of acquiring information does not seem to be purely azimuthal.

The authors have been focussing on the problem of pure bearing estimation without temporal measurements of the target heading angle by a single station, and several analytical methods have been given based on the condition that a stationary single station is able to acquire the target's bearing on the target's travelling path several times in a row at equal intervals [10-12].

The authors first gave the Cartesian Coordinate Method [10]. From the equations of the moving trajectory line of the target and the equation of the azimuthal line between the station and the target, an analytical representation of the target heading angle is derived from only three consecutive orientations, independent of the detection of the time parameter [10]. Among the several existing methods, the measurement accuracy given by the Cartesian coordinate method is the worst.

Then, the authors gave a method to solve the target heading angle based on the tangent median relation [11]. Using the flight trajectory of the target to construct the baseline of the one-dimensional double base array, the heading angle of the target can be given by the relationship between the heading angle and the front angle and the angle of arrival.

Subsequently, the authors further found that the analytical representation of the target heading angle can be given in a simpler way by directly using the trigonometric equations [12].

In this paper, on the basis of the trigonometric method, a new method of pure bearing estimation of target heading is given again based on the single-base midpoint direction finding formula. The single-base midpoint direction finding formula is a formula derived by the authors on the basis of the study of the linear solution of the three-station path difference localisation, which shows that: for a single-base array, after obtaining the path difference between the radial distances between the array elements at the two ends of the single-base line and the target to be measured, the target angle of arrival at the midpoint of the base line can be approximated by using the ratio of the path difference to the length of the base line. Compared with the trigonometric method, which is to find the heading angle directly from the ratio of the trigonometric equations, the new method proposed in this paper is to find the heading angle indirectly from the single-base mid-point direction finding equation, but the single-base mid-point direction finding equation is in fact also a ratio.

II. Derivation of the formula

As shown in Fig. 1, a stationary single station is used as the origin of the right-angle coordinate system, assuming that the detected target moves along the straight line AB at a uniform speed, and assuming that the stationary single station is able to obtain the orientation information of the target three times consecutively with equal spacing on the target's moving path. According to the trigonometric relationship, using the sine function, respectively, there are:

$$(1)$$

$$(2)$$

Where: d is the baseline constructed from the target's trajectory; r_1 and r_2 are the radial distances between the target and the fixed station; φ_1 , φ_2 and φ_3 are the azimuths at which the fixed station detects the target; and α is the angle of arrival of the target relative to the fixed station.

Use the results of the trigonometric equations to construct the path difference:

$$(3)$$

The angle of arrival of the target with respect to a fixed station can be obtained by directly using the single-base midpoint ranging equation [13]:

$$(4)$$

Based on the above results can be solved:

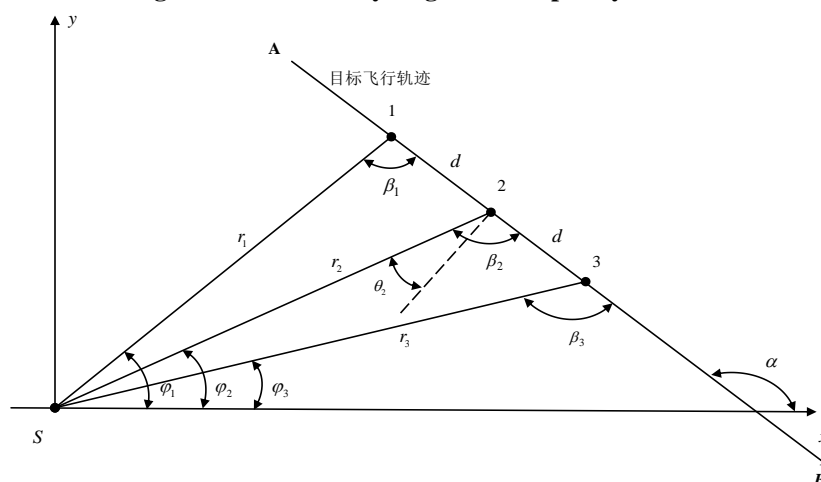
$$(5)$$

Obviously, the analytical formula for the angle of arrival of the target with respect to the fixed detection station is independent of the unknown quantity d . Further the heading angle of the target can be derived from the inside/outside angle relationship:

$$(6)$$

Where: θ_2 is the heading angle of the target; β_1 is the front angle of the moving target.

Fig. 1 Schematic diagram of a stationary single-station purely azimuthal tracking target



III. Simulation

The distance between the target and the detection station r_1 , the relative azimuth angle φ_1 , and the target's travelling distance d are preset, and the initial front angle of the target is varied within the range of α . Subsequently, the remaining radial distances r_2 and r_3 , and the relative azimuth angles φ_2 and φ_3 can be solved sequentially from the trigonometric relations, and the theoretical values of the target heading angles θ_2 can be obtained directly from the preset theoretical parameters from the internal and external angle relations:

$$(7)$$

Simulate measured values with calculated values:

$$(8)$$

The relative computational error is obtained by comparing the computational solution (8) with the theoretical value of the heading angle (7). The parameter values taken for the calculation are: (1) target isometric distance travelled: d ; (2) radial distance of the probe station: r_1 ; and (3) starting azimuth of the station: φ_1 .

The relative computational errors of heading angles at different starting azimuths are given in Fig. 2, and based on the geometric configuration of Fig. 1, the greater the starting azimuth the better the computational accuracy.

Fig. 2 Relative computational errors of heading angles at different starting azimuths

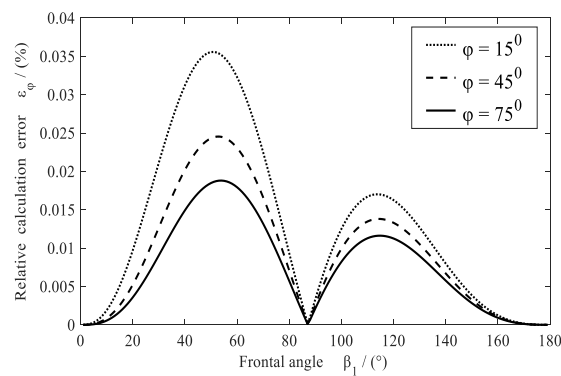


Figure 3 gives the relative calculation error of heading angle at different travelling distances, the smaller the travelling distance the better the calculation accuracy.

Fig. 3 Relative calculation error of heading angle for different travelling distances

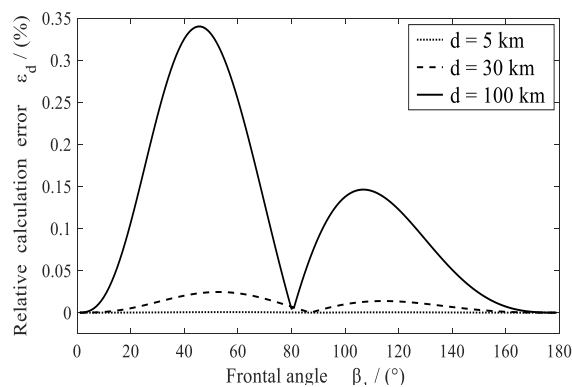
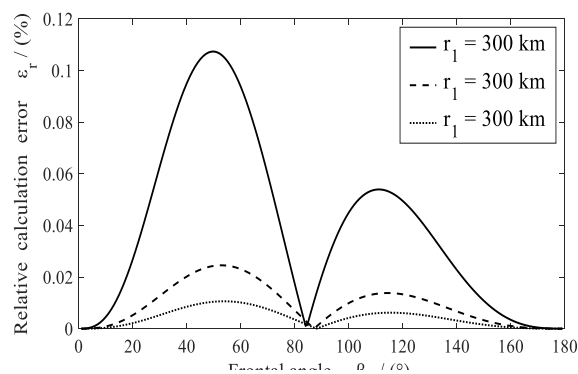


Figure 4 gives the relative computational errors of the heading angles at different radial distances, with better computational accuracy at larger radial distances.

Fig. 4 Relative computational errors of heading angles at different radial distances



Simulations show that the calculation accuracy is proportional to the radial distance and inversely proportional to the distance travelled.

IV. Error analysis

According to the theory of error estimation and synthesis, the total measurement error of the heading angle resulting from the angular measurement is:

$$(9)$$

Where: is the root-mean-square (RMS) value of the error in the direction-finding angle of the detection station.

Partial derivative

The partial differentials of the heading angle to the measured angle are respectively:

$$\frac{\partial \alpha}{\partial \varphi_1} = -0.5 \left(\frac{1}{\cos(\varphi_1 - \varphi_2) \cdot \sin(\varphi_2 - \varphi_3)} + \frac{1}{\sin(\varphi_1 - \varphi_2) \cdot \cos^2(\varphi_2 - \varphi_3)} \right)$$

$$\frac{\partial \alpha}{\partial \varphi_2} = 1 + 0.5 \left\{ \frac{1}{\cos(\varphi_2 - \varphi_3) \left[1 + \frac{1}{\sin^2(\varphi_2 - \varphi_3)} \right]} - \frac{1}{\cos(\varphi_1 - \varphi_2) \left[1 + \frac{1}{\sin^2(\varphi_1 - \varphi_2)} \right]} \right\}$$

$$\frac{\partial \alpha}{\partial \varphi_3} = -0.5 \left\{ \frac{1}{\cos(\varphi_2 - \varphi_3) \left[1 + \frac{1}{\sin^2(\varphi_2 - \varphi_3)} \right]} + \frac{1}{\cos(\varphi_1 - \varphi_2) \left[1 + \frac{1}{\sin^2(\varphi_1 - \varphi_2)} \right]} \right\}$$

Error curves

Using the same parameter values as in the simulation, the root-mean-square (RMS) value of the measurement angle error is taken as σ_α . The simulation shows that the starting azimuth angle has no effect on the measurement error.

Figure 5 gives the measurement error of the heading angle at different travelling distances, with a large measurement error at small travelling distances.

Figure 6 gives the measurement errors of the heading angles at different radial distances, with large measurement errors at larger radial distances.

Fig. 5 Measurement error of heading angle at different travelling distances

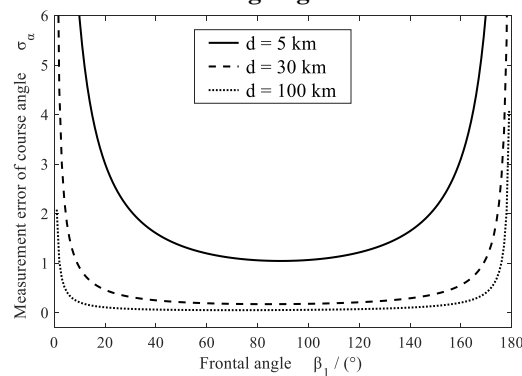
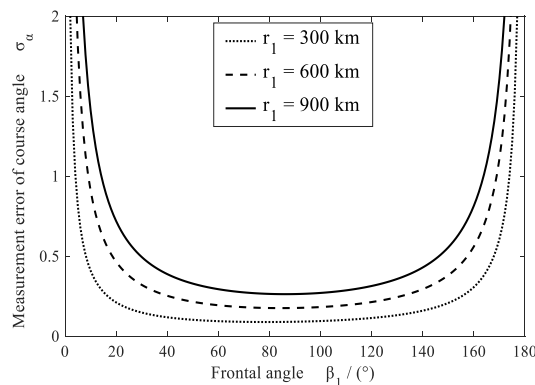


Fig. 6 Measurement error of heading angle at different radial distances



Comparison with trigonometry

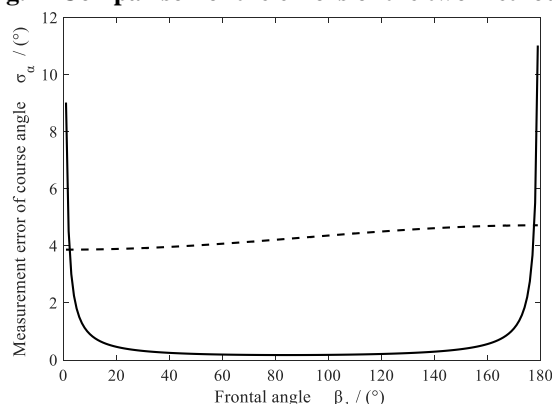
Literature [12] gives an analytical representation of the target heading angle in a simple mathematical expression by directly using trigonometric relations to give a definite solution equation :

(10)

Figure 7 compares the angular measurement errors of the two methods, and the results show that the single-base midpoint orientation method has better measurement accuracy than the trigonometric method. At this time, the authors cannot explain why the measurement accuracy can become better with the introduction of a single-base mid-point orienteering solution, but one thing that can be determined is that the introduction of a single-base mid-point orienteering solution further increases the constraints of the geometric problem.

In fact, without considering the relationship between internal and external angles, although two triangles are constructed using the target's trajectory, the conditions for solving the triangles do not seem to be fulfilled based only on a baseline formed by the target's trajectory, and the angles obtained by the fixed detection station. The resulting equations for the definite solution of the trigonometric functions appear to be unstable. With the introduction of the single-base midpoint vectors, an angular quantity is added which formally satisfies the conditions for the solution of two angles on one side of a triangle.

Fig. 7 Comparison of the errors of the two methods



V. Conclusion

The present article is the best method in terms of measurement accuracy among the four pure heading angle solving methods independent of time parameters provided by the author to date.

However, the purely azimuthal methods of solving for heading angles given in the studies to date have been analysed based on the assumption of equal baseline lengths. In fact the baseline length constructed from the target trajectory is unknown and needs to be eliminated by listing more conditions. In all four methods so far the baseline length is eliminated.

The Cartesian method is based on the derivation of coordinate variables, where the length of the baseline is not directly visible and the unknown Cartesian variables are eventually eliminated by solving the equation. In the tangent theorem's method of solving for heading angles, the formula is related to the angle only. In both the trigonometric method and the single-base midpoint direction finding method, the equations are directly related to the baseline length, but the baseline length is naturally cancelled out.

The main key that affects the accuracy of the measurement is how to detect the baseline length equidistantly, and in the way based on three times angle detection, the fixed single station can not achieve the equal baseline length detection either by using the equal time difference or the equal angle difference. This is the main reason that affects the engineering application of the research results.

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